Supplementary Information For "An Indirect-Reciprocity Game Theoretic Framework for Device-to-Device Multicast"

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I. Proof of Theorem 1

Theorem 1: There exists a unique stationary reputation distribution of the whole population for any given optimal action rule, and the stationary reputation distribution is $\boldsymbol{P_T}$'s eigenvector with the corresponding eigenvalue one.

Proof: Let $f(\lambda)$ denote the characteristic polynomial of matrix $\boldsymbol{P_T}$, which is given by

$$
\begin{aligned}
f(\lambda) &= |\lambda \boldsymbol{I} - \boldsymbol{P_T}| \\
&= \begin{vmatrix} \lambda - P_{T_{1,1}} & -P_{T_{1,2}} & \cdots & -P_{T_{1,L+S}} \\ -P_{T_{2,1}} & \lambda - P_{T_{2,2}} & \cdots & -P_{T_{2,L+S}} \\ \vdots & \vdots & \ddots & \vdots \\ -P_{T_{L+S,1}} & -P_{T_{L+S,2}} & \cdots & \lambda - P_{T_{L+S,L+S}} \end{vmatrix}.
\end{aligned} \tag{1}
$$

With the property of determinant, if the columns from the second to the last of $\lambda \boldsymbol{I} - \boldsymbol{P_T}$ are added to the first one, the determinant is unchanged. Besides, the sum of each row in $\boldsymbol{P_T}$ is equal to 1, i.e., $\forall i \in \{1, 2, ..., L+S\}$, we have,

$$\sum_{j=1}^{L+S} P_{T_{i,j}} = 1. \tag{2}$$

Hence, the characteristic polynomial of matrix $\boldsymbol{P_T}$ can be rewritten as

$$\begin{aligned} f(\lambda) &= |\lambda \boldsymbol{I} - \boldsymbol{P_T}| \\ &= \begin{vmatrix} \lambda - 1 & P_{T_{1,2}} & \cdots & P_{T_{1,L+S}} \\ \lambda - 1 & \lambda - P_{T_{2,2}} & \cdots & P_{T_{2,L+S}} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda - 1 & P_{T_{L+S,2}} & \cdots & \lambda - P_{T_{L+S,L+S}} \end{vmatrix}. \end{aligned} \tag{3}$$

Noticing that the elements in the first column of (3) are all $\lambda - 1$, we can conclude that $\lambda = 1$ is a root of the characteristic equation. Hence, $\boldsymbol{P_T}$ and its transpose must have an eigenvalue of 1, and the eigenvector corresponding to this eigenvalue is a solution to (12) in [1]. ■

II. Proof of Theorem 2

Theorem 2: Given the SL s, if g and c_e satisfy

$$g\beta W(1-\xi)\tilde{\eta}\triangle_1(s,l) + \delta(1-\xi)\triangle_2(s,l) \geq c_e L, \ \forall l \in \mathcal{L}, \tag{4}$$

the optimal action rule for the HN can be found as follows

$$A^* = \begin{pmatrix} \overbrace{L+1 \ \cdots \ L+1}^{D_0} & \overbrace{L \ \cdots \ L}^{D_1} & \cdots & \overbrace{3 \ \cdots \ 3}^{D_{L-2}} & \overbrace{2}^{D_{L-1}} \\ L+1 \ \cdots \ L+1 & L \ \cdots \ L & \cdots & 3 \ \cdots \ 3 & 2 \\ \vdots \quad \vdots \quad \vdots & \vdots \quad \vdots \quad \vdots & \vdots & \vdots \quad \vdots \quad \vdots & \vdots \\ L+1 \ \cdots \ L+1 & L \ \cdots \ L & \cdots & 3 \ \cdots \ 3 & 2 \end{pmatrix} \tag{5}$$

Here, $W = w_b\bar{B}\bar{N} + w_e\bar{P}$, $\triangle_1(s,l) = \min\limits_{l'} \sum\limits_{k=s}^{s+L} k(G_{l,k-s+1} - G_{l',k-s+1})$, $\triangle_2(s,l) = \min\limits_{l'} \sum\limits_{k=1}^{L+1} (G_{l,k} - G_{l',k})U_{k+s-1,:}\boldsymbol{q}^T$, and $\tilde{\eta} = (\sum_{k=1}^{L+S} k\eta_k - 1)^{-1}$.

Proof: Suppose the SL is s and the LDM is within the ones where only the outermost $L-l$ ($l \in \mathcal{L}'$) rings are not distributed with RNs. To serve all the RNs with ratio level $L+1$ and gain the highest immediate reputation value $\Omega_{s,L+1} = L+s$, the HN's action should be $a_{i,j} = l$, $\forall i,j$. According to (9), (14) and (16) of [1], we have

$$\boldsymbol{r}(a_{i,j},s) = (1-\xi)\sum_{m=1}^{L+1} e_{\Omega_{s,m}} G_{l,m} + \xi e_i, \tag{6}$$

$$t(a_{i,j},s) = \frac{(1-\xi)\sum_{k=s}^{s+L} kG_{l,k-s+1} + \xi i - 1}{\sum_{k=1}^{L+S} k\eta_k - 1}, \tag{7}$$

and

$$U(a_{i,j}, s) = -sd_{BH} - c_e l + g\beta W \frac{(1-\xi)\sum_{k=s}^{s+L} kG_{l,k-s+1} + \xi i - 1}{\sum_{k=1}^{L+S} k\eta_k - 1} + \delta \sum_k \sum_m r_k(a_{i,j}, s) U_{k,m} q_m$$
$$= -sd_{BH} - c_e l + g\beta W \frac{(1-\xi)\sum_{k=s}^{s+L} kG_{l,k-s+1} + \xi i - 1}{\sum_{k=1}^{L+S} k\eta_k - 1} + \delta\xi U_{i,:}\boldsymbol{q}^T + \delta(1-\xi)\sum_{k=s}^{s+L} G_{l,k-s+1} U_{k,:}\boldsymbol{q}^T \quad (8)$$

where $W = w_b\bar{B}\bar{N} + w_e\bar{P}$, $\boldsymbol{q} = [q_1, ..., q_D]$, and $\boldsymbol{q}^T$ is the transpose of $\boldsymbol{q}$.

Obviously, the HN will not choose action $a'_{i,j} > a_{i,j}$, since such an action will consume the HN more energy and will not increase the ratio of served RNs and the reputation value any more. Instead, the HN may choose to save energy by using less power to server only partial of the RNs. Suppose the other actions the HN may take are $a'_{i,j} = l'$. Similarly, we have

$$\boldsymbol{r}(a'_{i,j}, s) = (1-\xi)\sum_{m=1}^{L+1} e_{\Omega_{s,m}} G_{l',m} + \xi e_i, \quad (9)$$

$$t(a'_{i,j}, s) = \frac{(1-\xi)\sum_{k=s}^{s+L} kG_{l',k-s+1} + \xi i - 1}{\sum_{k=1}^{L+S} k\eta_k - 1}, \quad (10)$$

and

$$U(a'_{i,j}, s) = -sd_{BH} - c_e l' + g\beta W \frac{(1-\xi)\sum_{k=s}^{s+L} kG_{l',k-s+1} + \xi i - 1}{\sum_{k=1}^{L+S} k\eta_k - 1}$$
$$+ \delta\xi U_{i,:}\boldsymbol{q}^T + \delta(1-\xi)\sum_{k=s}^{s+L} G_{l',k-s+1} U_{k,:}\boldsymbol{q}^T. \quad (11)$$

For $a_{i,j} = l$ to be the optimal action, the following inequation

$$U(a_{i,j}, s) \geq U(a'_{i,j}, s), \quad (12)$$

should be held, which is equivalent to

$$-c_e(l-l') + g\beta W \frac{(1-\xi)\sum\limits_{k=s}^{s+L} k(G_{l,k-s+1} - G_{l',k-s+1})}{\sum_{k=1}^{L+S} k\eta_k - 1} + \delta(1-\xi)\sum_{k=s}^{s+L}(G_{l,k-s+1} - G_{l',k-s+1})U_{k,:}\boldsymbol{q}^T$$
$$= -c_e(l-l') + g\beta W \frac{(1-\xi)\sum\limits_{k=1}^{L+1} (k+s-1)(G_{l,k} - G_{l',k})}{\sum_{k=1}^{L+S} k\eta_k - 1} + \delta(1-\xi)\sum_{k=1}^{L+1}(G_{l,k} - G_{l',k})U_{k+s-1,:}\boldsymbol{q}^T \geq 0. \quad (13)$$

Let $\triangle_1(s,l) = \min\limits_{l'} \sum\limits_{k=s}^{s+L} k(G_{l,k-s+1} - G_{l',k-s+1})$, $\triangle_2(s,l) = \min\limits_{l'} \sum\limits_{k=1}^{L+1}(G_{l,k} - G_{l',k})U_{k+s-1,:}\boldsymbol{q}^T$, and $\tilde{\eta} = \frac{1}{\sum_{k=1}^{L+S} k\eta_k - 1}$, for (14) to be held, we should have

$$-c_e(l-l') + g\beta W(1-\xi)\tilde{\eta}\triangle_1(s,l) + \delta(1-\xi)\triangle_2(s,l) \geq 0, \quad (14)$$

and further have

$$g\beta W(1-\xi)\tilde{\eta}\triangle_1(s,l)+\delta(1-\xi)\triangle_2(s,l)\geq c_e l. \tag{15}$$

■

REFERENCES

[1] Y. Chen, B. Zhang, M. Wang, T. Xu, and Z. Han,“An Indirect-Reciprocity Game Theoretic Framework for Device-to-Device Multicast”, submitted to GLOBECOM 2019.