# Supplementary Information for "Contract-Based Cache Renting Mechanism in UAV-Assisted 5G Networks"

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## I. Proof of Lemma 1

***Lemma* 1.** The IR and IC constraints in problem (17) in [1] can be reduced as follows

(i) $\varphi_K < ... < \varphi_1$,

(ii) $T_K \leq \theta_K \varphi_K Q$, and

(iii) $\theta_{k-1}(\varphi_k - \varphi_{k-1})Q < T_k - T_{k-1} < \theta_k(\varphi_k - \varphi_{k-1})Q$, $\forall k \in \{2, ..., K\}$.

*Proof:* Notice that (i) and (ii) are the sufficient and necessary conditions of the IR constraints, while (i) and (iii) are the sufficient and necessary conditions of the IC constraints.

(i) Suppose the contract items for the type-$\theta_k$ CP and the type-$\theta_l$ CP, $k \neq l$, are $(\varphi_k, T_k)$ and $(\varphi_l, T_l)$, respectively. According to the IC constraints, we have

$$\theta_k \varphi_k Q - T_k > \theta_k \varphi_l Q - T_l, \tag{1}$$

and

$$\theta_l \varphi_l Q - T_l > \theta_l \varphi_k Q - T_k. \tag{2}$$

By combining (1) and (2), we can obtain

$$(\theta_k - \theta_l)(\varphi_k - \varphi_l)Q > 0. \tag{3}$$

Hence, $\varphi_k > \varphi_l$ if and only if $\theta_k > \theta_l$, implying more cache space should be leased to the CP whose videos are more valuable. Further, together with the constraint $0 \leq \varphi_k \leq 1$, for $\theta_K < ... < \theta_1$, we have $0 \leq \varphi_K < ... < \varphi_1 \leq 1$.

(ii) Since $\theta_k > \theta_l$, then we have $U_k = \theta_k \varphi_k Q - T_k > \theta_k \varphi_l Q - T_l > \theta_l \varphi_l Q - T_l = U_l$, implying CPs of higher types gain more profit from cache renting.

Further, for $\theta_K < ... < \theta_1$, we have $U_1 > ... > U_K$. Hence, if $U_K \geq 0$, there is $U_k \geq 0, \forall k < K$. In such case, IR constraints of K types CPs can be reduced to $U_K \geq 0$, i.e., $T_K \leq \theta_K \varphi_K Q$.

(iii) Local downward IC constraints (LDIC): For $\theta_K < ... < \theta_1$, if the IC constraints hold between any $k$-type and ($k$-1)-type CP, we have $U_k(\varphi_k, T_k) > U_k(\varphi_{k-1}, T_{k-1})$ and $U_{k-1}(\varphi_{k-1}, T_{k-1}) > U_{k-1}(\varphi_{k-2}, T_{k-2})$, i.e.,

$$\theta_k \varphi_k Q - T_k > \theta_k \varphi_{k-1} Q - T_{k-1}, \tag{4}$$

and

$$\theta_{k-1} \varphi_{k-1} Q - T_{k-1} > \theta_{k-1} \varphi_{k-2} Q - T_{k-2}, \tag{5}$$

Since for $\theta_{k-1} < \theta_{k-2}$, there is $\varphi_{k-1} < \varphi_{k-2}$. Then with $\theta_k < \theta_{k-1}$ and (5), we have

$$\theta_k(\varphi_{k-1} - \varphi_{k-2})Q > \theta_{k-1}(\varphi_{k-1} - \varphi_{k-2})Q > T_{k-1} - T_{k-2}. \tag{6}$$

Moreover, combing (4) and (6) we have

$$\theta_k \varphi_k Q - T_k > \theta_k \varphi_{k-2} Q - T_{k-2}, \tag{7}$$

i.e., $U_k(\varphi_k, T_k) > U_k(\varphi_{k-2}, T_{k-2})$, indicating that the downward IC constraints also hold between $k$-type and ($k$-2)-type CP.By induction method, we can conclude that if the downward IC constraints hold between the $k$-type and ($k$-1)-type CP, they also hold between the $k$-type and all of the ($k$-2),...,1-type CPs.

Local upward IC constraints (LUIC): if the IC constraints hold between any $k$-type and ($k$+1)-type CP, we have $U_k(\varphi_k, T_k) > U_k(\varphi_{k+1}, T_{k+1})$ and $U_{k+1}(\varphi_{k+1}, T_{k+1}) > U_{k+1}(\varphi_{k+2}, T_{k+2})$, i.e.,

$$\theta_k \varphi_k Q - T_k > \theta_k \varphi_{k+1} Q - T_{k+1}, \tag{8}$$

and

$$\theta_{k+1} \varphi_{k+1} Q - T_{k+1} > \theta_{k+1} \varphi_{k+2} Q - T_{k+2}, \tag{9}$$

Since for $\theta_{k+1} > \theta_{k+2}$, there is $\varphi_{k+1} > \varphi_{k+2}$. Then with $\theta_k > \theta_{k+1}$ and (9), we have

$$\theta_k(\varphi_{k+1} - \varphi_{k+2})Q > \theta_{k+1}(\varphi_{k+1} - \varphi_{k+2})Q > T_{k+1} - T_{k+2}. \tag{10}$$

Moreover, from (8) and (10) we have

$$\theta_k \varphi_k Q - T_k > \theta_k \varphi_{k+2} Q - T_{k+2}, \tag{11}$$

i.e., $U_k(\varphi_k, T_k) > U_k(\varphi_{k+2}, T_{k+2})$, indicating that the upward IC constraints also hold between $k$-type and ($k$+2)-type CP.

By induction method, we can conclude that if the upward IC constraints hold between the $k$-type and ($k$+1)-type CP, they also hold between the $k$-type and all of the ($k$+2),...,K-type CPs.

Therefore, the IC constraints for any $k$-type CP can be reduced to (4) and (8), i.e.,

$$\theta_{k-1}(\varphi_k - \varphi_{k-1})Q < T_k - T_{k-1} < \theta_k(\varphi_k - \varphi_{k-1})Q. \quad (12)$$

■

## II. Proof of Theorem 1

*Proof:* First, (19) in [1] satisfies $T_k^* \leq T_{k+1}^* + \theta_k(\varphi_k - \varphi_{k+1})Q$, i.e., the IR and IC constraints, according to Lemma 1.

Next, we prove by contradiction that the renting payment in (19) in [1] maximizes the MNO's utility defined in (18) in [1]. Given the fixed cache allocation, the utility of the MNO is decided by $\sum_{k=1}^{K} \pi_k T_k$. Suppose that there exists another feasible payment $\{\widehat{T}_k, \forall k \in \mathcal{K}\}$, which can make $\sum_{k=1}^{K} \pi_k T_k$ bigger than the payment defined in (19) in [1]. Thus, there is at least one payment $\widehat{T}_k > T_k^*$ for one type $\theta_k$. If $k$=$K$, then $\widehat{T}_K > T_K^*$. Since $T_k^* = \theta_K \varphi_K Q$, then $\widehat{T}_k > \theta_K \varphi_K Q$, which violates the IR constraints for type $\theta_K$. If $k < K$, since $\{\widehat{T}_k, \forall k \in \mathcal{K}\}$ must satisfy the IC constraints: $\theta_k \varphi_k Q - \widehat{T}_k > \theta_k \varphi_{k+1} Q - \widehat{T}_{k+1}$ or $\widehat{T}_k < \widehat{T}_{k+1} + \theta_k(\varphi_k - \varphi_{k+1})Q$. By substituting $T_k^* = T_{k+1}^* + \theta_k(\varphi_k - \varphi_{k+1})Q$ into this equality, we have $\varphi_{k+1} > T_{k+1}^*$ By the induction method, we have $\widehat{T}_K > T_K^*$, which contradicts with the IR constraint for type $\theta_K$, implying such a $\{\widehat{T}_k, \forall k \in \mathcal{K}\}$ does not exist. ■

## III. Proof of Theorem 2

*Proof:* The payment in (19) in [1] can be re-written as

$$T_k^* = T_K^* + \sum_{i=k}^{K} \Delta_i, \quad (13)$$

where $\Delta_i = 0$ when $k = K$ and $\Delta_i = \theta_k(\varphi_k - \varphi_{k+1})Q$ when $0 < i < K$.

Substituting (13) into (18) in [1] and removing the monotonicity condition and integer constraints of videos cached at the MBS and UAVs, we have a relaxed optimal problem as

$$\max_{\{\varphi_k, \forall k \in \mathcal{K}\}} \sum_{k=1}^{K} Z_k(\varphi_k) \quad (14)$$
$$s.t. \quad (15) \text{ in [1], and } 0 \leq \varphi_k \leq 1,$$

where $Z_k(\varphi_k) = \pi_k \theta_k \varphi_k Q + \Lambda_k \sum_{i=1}^{k-1} \pi_i - \pi_k C(\varphi_k)$ and $\Lambda_k = \varphi_k Q(\theta_k - \theta_{k-1}), \ \forall k \in \{2, ..., K\}$ and $\Lambda_k = 0$ when $k$=1.

obviously, all the $Z_k(\varphi_k), \forall k \in \mathcal{K}$, and the constraints in (14) are the convex function of $\varphi_k$. The the Lagrangian function of (14) can be constructed as

$$U = \sum_{k=1}^{K} Z_k(\varphi_k) + \lambda(\sum_{k=1}^{K} \pi_k \varphi_k - 1) + \sum_{k=1}^{K} \lambda_k \varphi_k, \quad (15)$$

where $\lambda$ and $\lambda_k$ are the lagrange multipliers.

Setting the first order partial derivative of $\varphi_k$, $\lambda$ and $\lambda_k$, $k \in \mathcal{K}$, to zero, we have the KKT conditions as shown in (21) in [1] and solve it, the solution of (14), i.e.,$\widehat{\varphi}_k^*$, $k \in \mathcal{K}$, can be obtained.

Notice that if $\widehat{\varphi}_k^*$, $k \in \mathcal{K}$, satisfies the monotonicity condition, it is also the solution to (18) in [1], i.e., $\varphi_k^* = \widehat{\varphi}_k^*$, $k \in \mathcal{K}$. Nevertheless, we should find the infeasible subsequences of $\widehat{\varphi}_k^*$, such as $\{\widehat{\varphi}_i^*, \varphi_{i+1}^*, ..., \varphi_j^*\}$, if $\widehat{\varphi}_i^* < \widehat{\varphi}_{i+1}^* < ... < \widehat{\varphi}_j^*$. According to the Proposition 1 in [2], adjusted values satisfy $\varphi_i^* = \varphi_{i+1}^* = ... = \varphi_j^*$. Moreover, $\varphi_i^*, \varphi_{i+1}^*, ..., \varphi_j^*$ should keep the capacity constraints, i.e.,

$$\pi_i \varphi_i^* + \pi_{i+1} \varphi_{i+1}^* + ... + \pi_j \varphi_j^* = \pi_i \widehat{\varphi}_i^* + \pi_{i+1} \widehat{\varphi}_{i+1}^* + ... + \pi_j \widehat{\varphi}_j^*.$$

Hence, $\varphi_i^* = \varphi_{i+1}^* = ... = \varphi_j^* = \frac{\pi_i \widehat{\varphi}_i^* + \pi_{i+1} \widehat{\varphi}_{i+1}^* + ... + \pi_j \widehat{\varphi}_j^*}{\pi_i + \pi_{i+1} + ... + \pi_j}$.

On the other hand, for those points in $\{\widehat{\varphi}_k^*\}$ that follow the monotonicity constraint, we set directly that $\varphi_k^* = \widehat{\varphi}_k^*$. ■

## References


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